

Design of Sliding Mode Controller for Field Controlled DC Motor Under Parameter Uncertainties

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Received: 10th December 2017, Accepted: 17th January 2018, Published: 28th February 2018

Abstract

Designing the speed controller for a field controlled DC motor with uncertain parameters is a challenging task. Sliding Mode Control technique is widely used robust controller design methodology that takes care of uncertain parameters. Here, in this paper, first order sliding Mode controller is designed to regulate the speed of a DC motor using unit vector approach. The effectiveness of the control strategy in regulating the dynamics of the field controlled DC motor is verified in simulation in the MATLAB environment. Each parameter of this field controlled DC motor is varied by $\pm 50\%$ and performed simulations.

Keywords: Field controlled, First order SMC, Uncertainty, Unit vector Approach.

Introduction

DC motors are generally used in various industrial applications such as rolling mills, mining, traction, power plants, conveyor belts, paper industries etc... The usage of the DC motors spread from low horse power to multi-megawatt due to its high power, torque, speed ranges, high efficiency, and fast response [1]. In industrial process control sometimes it is necessary to be able to adjust motor's speed over a wide range with good speed and robust control. Therefore, it is an important task to design a controller for regulating the speed of DC motor.

Sliding mode control technique is inherently a robust control technique and it is used to regulate the speed of the DC motor control. The features of sliding mode control are order reduction, disturbance rejection, strong robustness. This sliding mode control is also known as variable structure control system which has been utilized in the designing of robust regulators, model reference systems, and adaptive schemes, tracking systems, state observers and fault detection schemes.

The procedural steps in designing SMC strategy are enumerated as follows.

- 1) Determining sliding surface that governs the system dynamics.
- 2) Defining the control law. The function of the control law is to attract the state trajectories towards the sliding surface [5] and also to maintain the trajectories on the sliding surface.

This paper is organized as follows. (i) Mathematical modelling aspects of the field controlled DC motor is discussed in the section II. (ii) Sliding mode Control law aspects are discussed in the section III. (iii) Control strategy derivation for field controlled DC motor is discussed in section IV. (iv) Simulation studies using MATLAB are discussed in section V. (iv) Conclusions are presented in section VI.

Modelling of Dc Motor

DC motors are used in two different control modes (i) armature control mode with fixed field current (ii) field control mode with fixed armature current. This paper discusses about speed control of the field controlled DC motor using sliding mode approach. The schematic diagram of a field controlled DC motor is shown in Fig. 1. The mathematical equations for field controlled DC motor [4] are given below step by step.

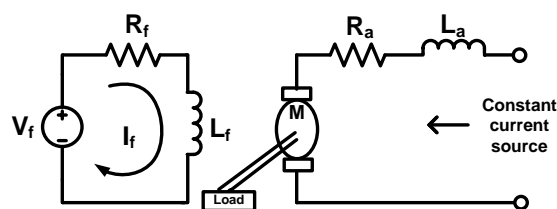


Fig . 1. Field controlled DC motor

The differential equation of this motor at the mechanical shaft is described using equation (1).

$$J_f \ddot{\theta}(t) + b_f \dot{\theta}(t) = T_f(t) \quad (1)$$

Where θ represents angular displacement of the motor shaft, J_f represents the equivalent moment of inertia of the motor and load referred to motor shaft, b_f is the equivalent viscous friction coefficient of motor and load referred to motor shaft.

The torque of a DC motor is proportional to product of flux and armature current and represented mathematically using the equation (3). Since the armature current is constant in this system, the torque is proportional to flux alone, but flux is proportional to field current i.e.

$$\phi \propto i_f \quad (2)$$

$$T_f = K_{tf} i_f \quad (3)$$

i_f is the field current.

The differential equation of the field circuit is given in equation (4).

$$L_f \frac{d}{dt} i_f(t) + R_f i_f(t) = V_f(t) \quad (4)$$

Where R_f and L_f are the field resistance (ohms) and field inductance (henrys) respectively. The $V_f(t)$, is applied voltage is taken to be the control variable. Define the error ratio for inductance as

$$\xi_L = \frac{L_0 - L_f}{L_f} \quad (5)$$

$|\xi_L| < 0.1$ since L_f is not known precisely, where L_0 is some nominal inductance value. Similarly, the inertia coefficient is also not known perfectly, hence

$$\frac{K_{tf}}{J_f} = \frac{K_{tf}}{J_0} + \xi_J \quad (6)$$

Where $|\xi_J| < 0.5$, and J_0 is some nominal value.

Finally assume that the friction effects are negligible, i.e. $b = 0$.

Using the variables $\theta, \dot{\theta}$ and i_a as state variables, the state equation of the field controlled DC motor system and its state space form are shown in equation (7).

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (7)$$

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{K_{tf}}{J_f} \\ 0 & 0 & -\frac{R_f}{L_f} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_f} \end{pmatrix}$$

Sliding Mode Control Design

Sliding Mode Control (SMC) is based on the concept of changing the structure of a controller in response to the changing state of a system with the aim of obtaining a desired response. A high-speed switching control action is used to switch between different structures, and the trajectory of the system is forced to move along a chosen manifold in the state space, called the switching surface [3]

(i) System Description:

An uncertain DC motor system is described using the equation (8).

$$\dot{x}(t) = Ax(t) + Bu(t) + f_u(t, x) + f_m(t, x, u) \quad (8)$$

The regular form representation is useful in designing the SMC. In regular form, the system matrix A can be partitioned and represented as, $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, and

$$B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

where A is $n \times n$

similarly A_{11} is $(n-1) \times (n-1)$, A_{12} is $(n-1) \times 1$, A_{21} is $1 \times (n-1)$, A_{22} is 1×1 and B_1 is $(n-1) \times 1$, B_2 is 1×1

$$\begin{aligned} \dot{x}_1(t) &= A_{11}x_1(t) + A_{12}x_2(t) + f_u(t, x) \\ \dot{x}_2(t) &= A_{21}x_1(t) + A_{22}x_2(t) + B_2u(t) + f_m(t, x, u) \end{aligned} \quad (9)$$

Here the functions $f_u(\cdot)$ and $f_m(\cdot)$ represents the unmatched and matched uncertainty components respectively[2]. These are unknown but bounded and satisfy the equation (10) where $k_1, k_2, k_3 \geq 0$ are known constants and $\alpha(\cdot)$ is a known function.

$$\begin{aligned} \|f_u(t, x)\| &\leq k_1 \|x\| + k_2 \\ \|f_m(t, x, u)\| &\leq k_3 \|u\| + \alpha(t, x) \end{aligned} \quad (10)$$

These k_1, k_2, k_3 and α values are obtained by equating equation (9) to zero.

(ii) Switching Function Description:

The switching surface is represented by using (11).

$$s = Sx \quad (11)$$

If the switching function matrix is partitioned and represented mathematically using equation (12).

$$S = [S_1 \quad S_2] \quad (12)$$

Then, the switching function can be expressed as

$$s(t) = S_1x_1(t) + S_2x_2(t) \quad (13)$$

During ideal sliding, $s(t) = 0$ hence, the sliding motion is described by (14).

$$S_1x_1(t) + S_2x_2(t) = 0 \quad (14)$$

Therefore expressing $x_2(t)$ in terms of $x_1(t)$ is

$$x_2(t) = -Mx_1(t) \quad (15)$$

By using equations (13) and (15) we can obtain switching function as

$$s(t) = S_2Mx_1(t) + S_2x_2(t) \quad (16)$$

Where $M = S_2^{-1}S_1$, $M = [m_1 \quad m_2]$.

(iii) Control Law Construction:

The sliding mode control law comprises two components[7]; a linear component to stabilise the nominal system; and the discontinuous component is to bring the state trajectories towards the switching surface or sliding surface [3]. The discontinuous component is represented mathematically using (17).

$$u = u_l + u_{nl} \quad (17)$$

Linear component can be obtained by differentiating equation (13) and substituting equations (9) and (15).

$$\begin{aligned} \dot{s}(t) &= S_2[M[A_{11} - A_{12}M] + A_{21} - A_{22}M]x_1(t) + \\ &S_2[MA_{12} + A_{22}]S_2^{-1}s(t) + \Lambda u(t) + S_2f_m(t, x, u) + S_1f_u(t, x) \end{aligned} \quad (18)$$

Where $\Lambda = S_2B_2$

According to sliding motion robustness properties i.e., in the absence of uncertainty[6] the linear component is obtained using equation (19).

$$u_1(t) = \Lambda^{-1} \left(-S_2 [M[A_{11} - A_{12}M] + A_{21} - A_{22}M] x_1(t) \right. \\ \left. \Lambda^{-1} (S_2 [MA_{12} + A_{22}] S_2^{-1} s(t) - \Phi) s(t) \right) \quad (19)$$

Φ represents a negative scalar which determines the linear rate of decay onto switching surface. Nonlinear component is defined to be

$$u_{nl} = -\rho(t, x) \Lambda^{-1} \frac{P_2 s(t)}{\|P_2 s(t)\|} \quad (20)$$

Where P_2 is a symmetrical positive definite matrix satisfying the Lyapunov equation $P_2 \Phi + \Phi^T P_2 = -I$ $\rho(t, x)$ is a scalar function which depends only on magnitude of the uncertainty (21).

$$\rho(t, x) \geq \frac{\|S_2\| \left(\|M\| (k_1 \|x(t)\| + k_2) + k_3 \|u_1(t)\| + \alpha(t, x) \right) + \gamma_2}{(1 - k_3 k(\Lambda) \|B_2^{-1}\|)} \quad (21)$$

Where $\gamma_2 > 0$ is a design parameter.

Λ is a scaled identity matrix, $k(\Lambda) = 1$.

For single – input case, the unit vector component is defined using equation (22)

$$\frac{P_2 s(t)}{\|P_2 s(t)\|} = \text{sgn}(s) \quad (22)$$

Smc for DC Motor

The modified state space form of the armature controlled DC motor after including the error ratio for inductance and inertia coefficient can be written as

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{K_{tf}}{J_f} \\ 0 & 0 & -\frac{R_f}{L_f} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_f} \end{pmatrix}$$

The regular form model of DC motor without uncertain components is given in equation (7) and using that $A_{11}, A_{12}, A_{21}, A_{22}, B_1$ and B_2 are obtained.

$$A_{11} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 0 \\ \frac{K_{tf}}{J_f} \end{pmatrix}, \quad A_{21} = \begin{pmatrix} 0 & 0 \end{pmatrix}, \quad A_{22} = \begin{pmatrix} -\frac{R_f}{L_f} \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 \\ L_f \end{pmatrix}$$

State equations of the field controlled DC motor with uncertain components is described using (23).

$$\dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -2\xi\omega_n x_2(t) - \omega_n^2 x_1(t) + f_u(t, x) \\ \dot{x}_3(t) = -\frac{R_f}{L_0} x_3(t) + \frac{1}{L_0} u(t) + f_m(t, x, u) \quad (23)$$

Where

$$f_u(t, x) = -\xi_J (m_1 x_1(t) - m_2 x_2(t)) \\ f_m(t, x, u) = -\frac{1}{L_0} \xi_L (R_f x_3(t) - u(t)) \quad (24)$$

Comparing with equation (10)

$$k_1 = \xi_J \|M\|, \quad k_2 = 0, \quad k_3 = \frac{\xi_L}{L_0} \\ \alpha(t, x) = -\frac{1}{L_0} \xi_L (R_f x_3(t)) \quad (25)$$

The switching function was obtained using pole-placement technique and needs a characteristic equation. The desired characteristic equation that characterizes the sliding motion is described by using a standard second order equation is given in (26).

$$\lambda^2 + 2\xi\omega_n \lambda + \omega_n^2 = 0 \quad (26)$$

For this uncertain DC motor formulate the null space dynamics using the equation (27).

$$\dot{x}_1(t) = (A_{11} - A_{12}M) x_1(t) \quad (27)$$

Comparing the characteristic equation of $(A_{11} - A_{12}M)$ with (26), the value of M is obtained and specified using (28).

$$M = \begin{bmatrix} \frac{J_0}{K_{tf}} \omega_n^2 & 2 \frac{J_0}{K_{tf}} \xi \omega_n \end{bmatrix} \quad (28)$$

The switching function 'S', is obtained by considering $S_2 = 1$ is defined by using (29).

$$S = \begin{bmatrix} \frac{J_0}{K_{tf}} \omega_n^2 & 2 \frac{J_0}{K_{tf}} \xi \omega_n & 1 \end{bmatrix} \quad (29)$$

Therefore, the switching surface for this field controlled DC motor was given by equation (30).

$$s(t) = \begin{bmatrix} \frac{J_0}{K_{tf}} \omega_n^2 & 2 \frac{J_0}{K_{tf}} \xi \omega_n & 1 \end{bmatrix} x \quad (30)$$

Substitute $s(t)$ in equation (19) to obtain linear part of the control law (equivalent control)

$$u_1(t) = -(SB)^{-1} (SA - \Phi S) x(t) \quad (31)$$

Substituted the respective values of field controlled DC motor (i.e., B) in the above equation (31) the linear component given in (32) is obtained.

$$u_1(t) = -L_0 S A x(t) + L_0 \Phi s(t) \quad (32)$$

And, the non linear component was obtained using equation (20) can be described using equations (33) and (34)

$$u_{nl}(t) = -\rho L_0 \operatorname{sgn}(s) \quad (33)$$

$$\text{Where } \rho = \frac{(|u_1(t)| + R_f |x_3(t)| + 5L_0 \|M\|^2 \|x(t)\| + 10L_0 \gamma_2)}{9L_0} \quad (34)$$

Results and Discussion

The field controlled DC motor parameters considered for simulation are given in Table I.

TABLE I: Field Controlled Dc Motor Parameters [2]

Name of the parameter	Value
Field Resistance (R_f)	100
Inductance (L_0)	22
Inductance (L_f)	20
Torque constant (K_{tf})	10
Moment of Inertia (J_f)	0.5
Moment of Inertia (J_0)	0.512
Natural frequency (ω_n)	2
Damping Ratio (ξ)	0.95
Constant (Φ)	-20

With the parameters listed in the table I, the field controlled DC motor is subjected to an initial condition [1 0 0], the history of state variables angular position, angular velocity, and field current in the interval [0 5], are respectively shown in Fig. 3, Fig. 4 and Fig. 5. When this field controlled DC motor system is subjected to the perturbation, these variables are returning to the original state.

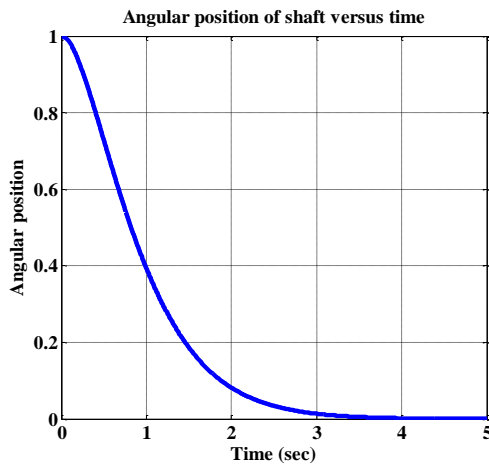


Fig. 3. Variation of Angular Position of shaft versus time

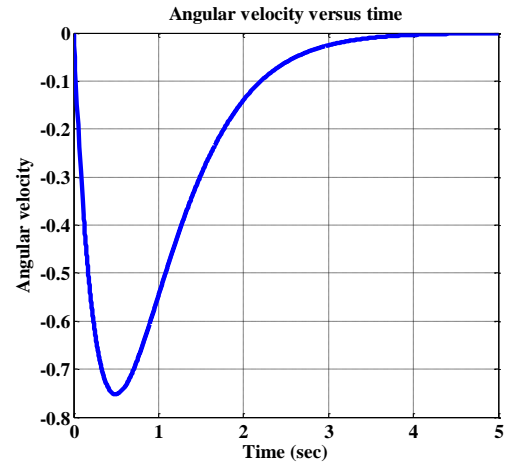


Fig . 4. Variation of Angular Velocity of shaft versus time

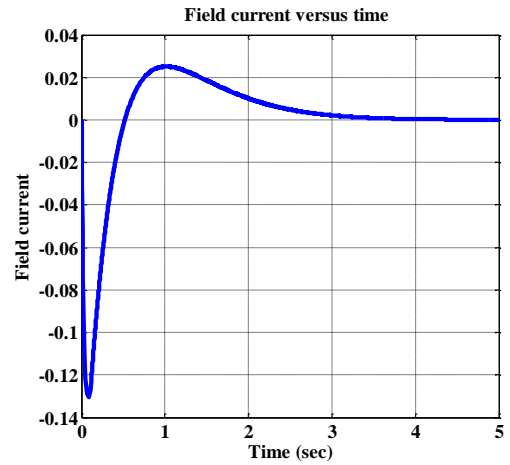


Fig. 5. Variation of Field current versus time

When this field controlled DC motor is subjected to the initial condition [1 0 0], the state trajectory in the three dimensional plane is shown in Fig. 6. It can be concluded that the state trajectory is reaching to the steady state operating point when it is perturbed.

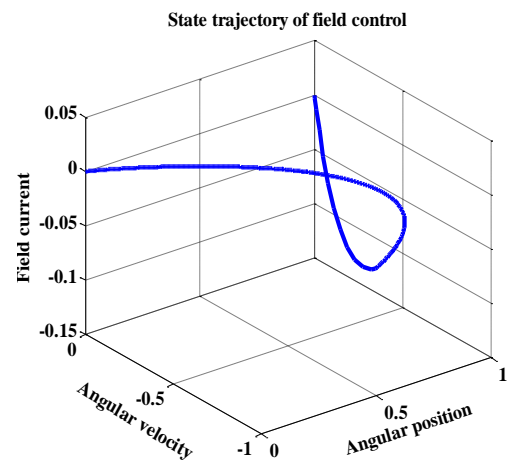


Fig. 6. State trajectories for field controlled DC Motor

The variation of switching surfaces of this field controlled DC motor when it is subjected to initial condition $[1 \ 0 \ 0]$ are shown in Fig. 7 and Fig. 8.

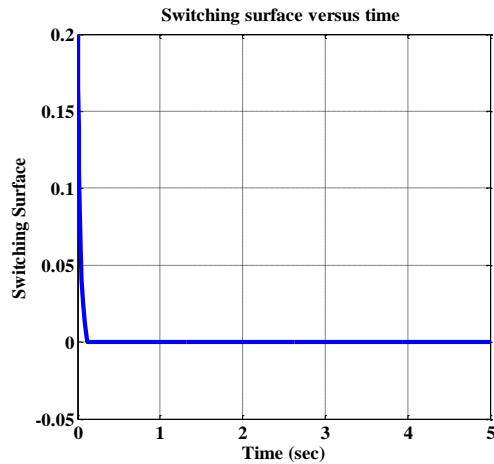


Fig. 7. Variation of Switching surface versus time

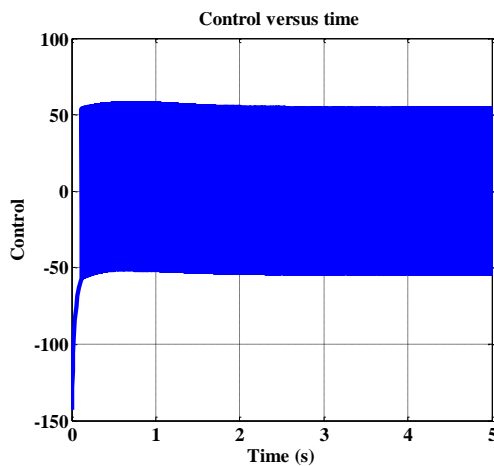


Fig. 8. Variation of Control action versus time

Each parameter given in Table I is increased by 50% and the same parameters are decreased by 50% then the simulated responses for angular position, angular velocity and field current are as shown in Fig. 9, Fig. 10 and Fig. 11 respectively when this system is subjected to a perturbation. It is observed that these variables are returning to the original state. When the parameters are decreased the deviations are more and taking more time to return to the original state.

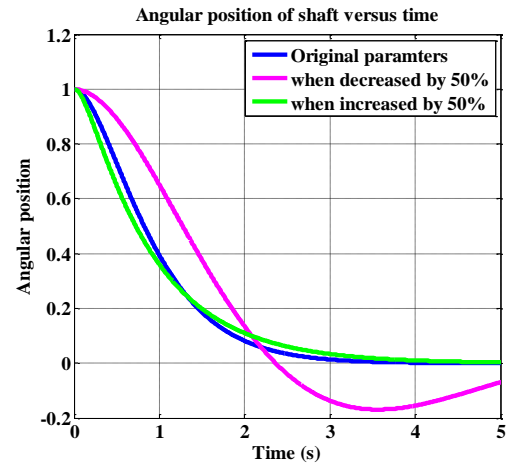


Fig. 9. Variation of Angular position with time under parameter variations

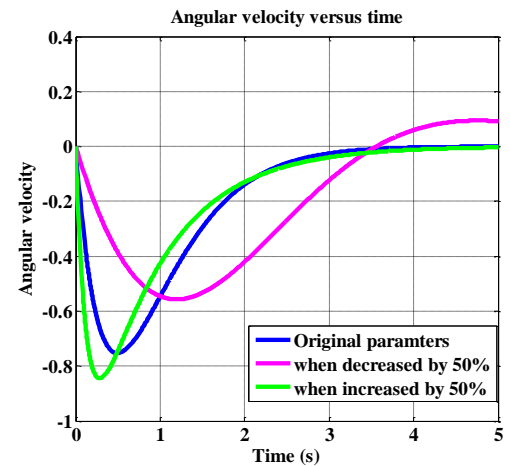


Fig. 10. Angular velocity variation with time under parameter variations

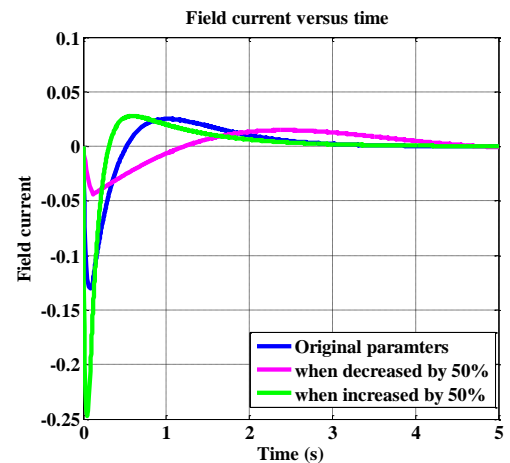


Fig. 11. Field current of DC motor versus time under parameter variations

This field controlled DC motor is subjected to the initial conditions $[1 \ 0 \ 0]$, the state trajectory in the three dimensional plane under variation of parameters is shown in Fig. 12. It can be concluded that the state

trajectory in all cases is reaching to the steady state operating point when it is perturbed.

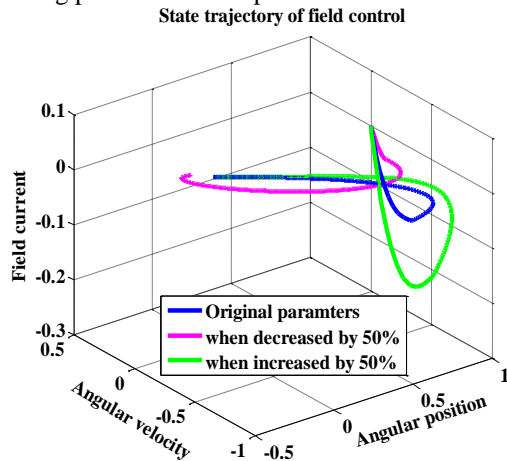


Fig. 12. State Trajectory for field control DC motor Under parameter variations

Switching surfaces of this system under parameter variations are shown in Fig. 13.

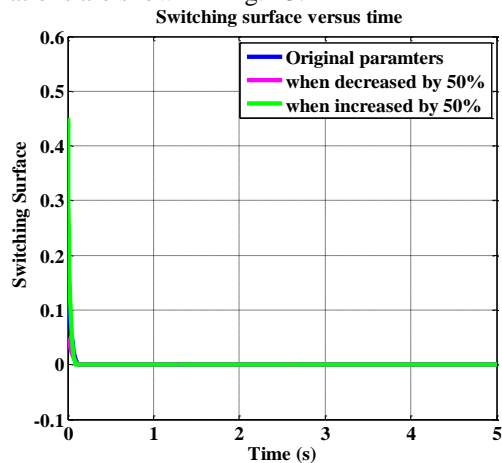


Fig . 13. Switching surface versus time under parameter variations

Control signal of this system with nominal parameters and with parameter variations are shown in Fig. 8 and Fig. 14 and Fig. 15. It is observed that this system needs higher control signal strength under parameter variations compared to nominal parameter case. With increased parameter set the signal strength is more compared to decreased parameter set.

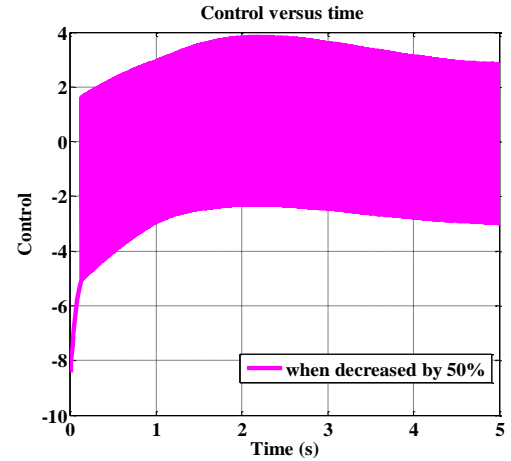


Fig. 14. Control action versus time with decreased parameters

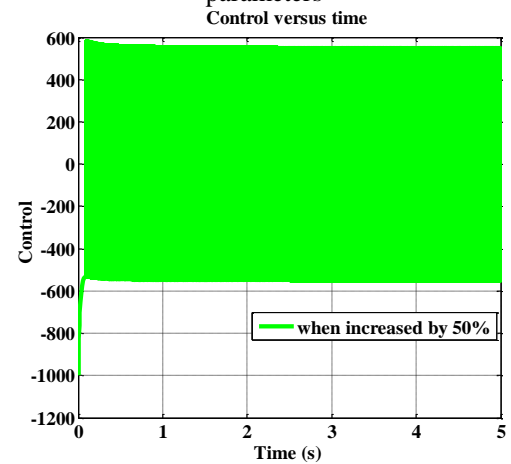


Fig. 15. Control action versus time with increased parameters

Conclusion

The paper dealt with an approach of first order sliding mode control applied to field controlled DC motor. Detailed modeling aspects of the field controlled DC motor are discussed. Detailed SMC design aspects under parameter variations are discussed. As each parameter of this field controlled system is increased by 50% and the same parameters are decreased by 50% then the variation of responses for the state variables angular position, angular velocity and field current are obtained. The history of all state variables with time are plotted under variations of parameters. Control signal and switching surface with time are plotted under parameter variations.

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