

Performance Evaluation of Hybrid Dual Tree Complex Wavelet Transform based Signal Denoising Technique

^{*1}D. Srinivasulu Reddy, ²M. Sadasiva, ³K. Gopi

¹SV Engineering College for Women, Tirupati, India,

^{2,3}Department of ECE, SV College of Engineering, Tirupati, India

Email: ^{*1}cnudega@gmail.com, ²m.sadasiva@gmail.com, ³kgr430@gmail.com

Received: 06th October 2017 Accepted: 14th November 2017, Published: 31st December 2017

Abstract

In this paper, the performance of the signal and denoising techniques using different types of Wavelet Transform techniques is evaluated based on the different performance metrics such as Signal to Noise Ratio(SNR). A new Complex Wavelet Transform technique called Hybrid Dual Tree Complex Wavelet Transform, has been proposed in this work, so as to achieve the better denoising performance over the existing Wavelet Transform based denoising techniques. As the proposed technique mixes all the features of the existing Wavelet Transform techniques, the best performance can be expected from this technique.

Keywords: Denoising, Discrete Wavelet Transform (DWT), Dual Tree Complex Wavelet Transform (DTCWT), Analytic Wavelet Transform (AWT), Hybrid Dual Tree Complex Wavelet Transform (HDTTCWT).

1. INTRODUCTION

It is quite common that any signal can be corrupted with noise during acquisition, transmission and other different sources of noise. To get the meaningful information from this noisy signal, it should be subjected to the pre-processing stage where the signal is made as suitable for further processing. The most significant step in the pre-processing stage is denoising of data which will greatly impact the processing stage performance in different applications viz., medical, defense etc.

As the performance of the time (spatial) domain based denoising methods viz., mean filter, median filter, Wiener filter etc. is not effective over the transform domain based denoising methods viz. Fourier Transform, Wavelet Transform etc., the transform domain based denoising methods have been opted always for reducing the noise present in the data. The different steps of the transform based denoising techniques are:

1. Compute the forward transform of the data
2. Treat the transform domain data using the optimum thresholding techniques
3. Compute the inverse transform of the treated transform domain data.

The Fourier Transform based denoising technique is not suitable for the non-stationary signals, hence its denoising performance cannot be comparable with Wavelet Transform based denoising methods which are suitable for non-stationary signals also.

In this work, a comparison between the performance of the different Wavelet Transform based denoising techniques ie evaluate. Moreover, Complex Wavelet Transform (CWT) based denoising techniques are emphasized over the Discrete Wavelet Transform (DWT) based denoising techniques because CWT can overcome the limitation of DWT such as shift sensitive, poor directionality and absence of phase information.

2. REVIEW OF EXISTING WAVELET TRANSFORM BASED DENOISING TECHNIQUES

In this section, different existing wavelet transform techniques such as Discrete Wavelet Transform (DWT), Dual Tree Complex Wavelet Transform (DTCWT), Analytic Wavelet Transform (AWT), based denoising techniques have been reviewed.

A. Discrete Wavelet Transform (DWT) based Signal Denoising Technique

Discrete Wavelet Transform of a signal decomposes a signal into approximation and detail coefficients which consists of low and high frequency components of the signal. As the noise is high frequency component, it presents in detail coefficients only, and hence detail coefficients should be treated by subjecting them to some thresholding rules. Wavelet decomposition and reconstruction of a signal x are shown in Fig 1; here h , g and \tilde{h} , \tilde{g} are Low pass and High pass FIR filters and their reconstruction filters respectively.

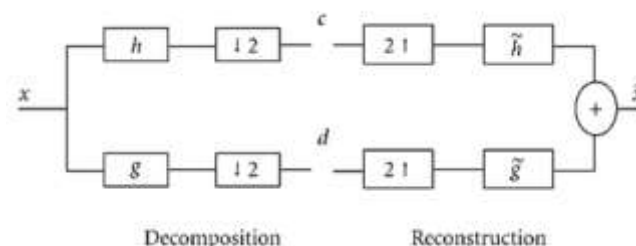


Fig 1. Wavelet Decomposition and Reconstruction of a signal

In the DWT based signal denoising technique, the given signal or image corrupted with noise is subjected to wavelet decomposition where the level of decomposition is decided by the amount of noise present in the signal or image. Of the approximation and detail coefficients of wavelet decomposition, the detail coefficients where the noise is predominant are considered for thresholding to reduce the noise. These modified detail coefficients and unaffected approximation coefficients are subjected to inverse wavelet transform (wavelet reconstruction) in order to get denoised signal or image.

Due to the limitations of DWT such as shift sensitive, poor directionality, the DWT based denoising techniques are not as effective- as Complex Wavelet Transform based Denoising Techniques. DWT based denoised techniques have been failed to reduce the noise present at sudden the transition of amplitude from one value to other in the signal due to its poor sensitivity to the abrupt transitions in a signal. However, the DWT senses the abnormal variation in a signal at the large number of decomposition levels, whereas the Complex Wavelet Transform provides high degree of sensitivity and hence CWT senses signal variations at earlier level decompositions itself.

Dual Tree Complex Wavelet Transform (DTCWT) based Denoising Technique

The limitations of the DWT such as shift sensitivity and poor directionality can be overcome by employing Complex Wavelet Transform to the signal for wavelet decomposition instead of DWT. The primary version of the Complex Wavelet Transform is Dual Tree Complex Wavelet Transform (DTCWT), it consists of two distinct wavelet decompositions where one tree of wavelet decomposition represents real valued wavelet coefficients and the other tree of wavelet decomposition represents the imaginary values of the wavelet coefficients. The DTCWT decomposition and reconstruction of the signal are shown in Fig 2 (a) and 2 (b) respectively.

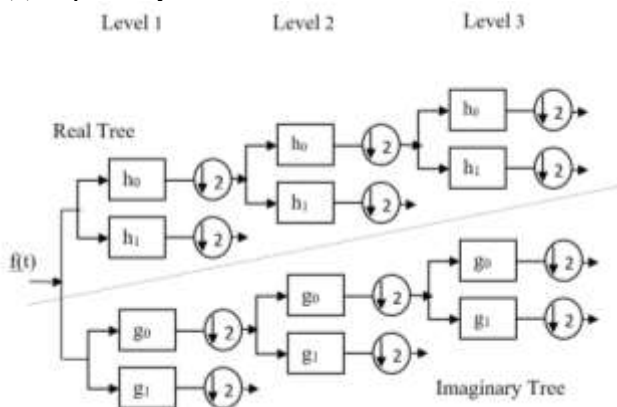


Fig 2 (a) DTCWT Decomposition of the signal f

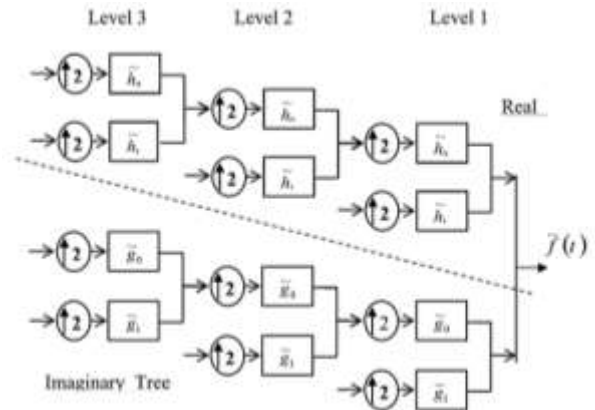


Fig 2 (b) DTCWT Reconstruction of \hat{f}

In the above figures, Fig 2 (a) and Fig 2 (b), f is the signal that is being undergone DTCWT decomposition using real tree (tree a) low pass and high pass FIR filters h_0 and h_1 and imaginary tree (tree b) low pass and high pass FIR filters g_0 and g_1 ; and \hat{f} is the signal reconstructed from the decomposition approximation and details, using real tree (tree a) low pass and high pass FIR

The scaling and wavelet functions of the real tree and imaginary tree are designed in such a way that the scaling and wavelet function of one tree form a Hilbert Transform pair with those of the other tree. The Low-pass and High-pass decomposition and reconstruction FIR filters of the real tree (tree a) and imaginary tree are designed by subject to the condition that wavelet and scaling functions of the tree a form Hilbert Transform pair with those of tree b.

Let $\phi_a(t)$, $\varphi_a(t)$ and $\phi_b(t)$, $\varphi_b(t)$ be the scaling and wavelet functions of tree a and tree b respectively, As $\varphi_a(t)$ and $\varphi_b(t)$, $\phi_a(t)$ and $\phi_b(t)$ and form Hilbert Transform pairs, that is,

$$\begin{aligned}\varphi_b(t) &= H\{\varphi_a(t)\} \text{ and} \\ \phi_b(t) &= H\{\phi_a(t)\} \dots\dots\dots (1)\end{aligned}$$

The decomposition and reconstruction Low pass and High pass filters are designed subject to the condition (1).

The Dual Tree Complex Wavelet Transform (DTCWT) based signal denoising technique involves the following three steps:

1. Decompose the given noisy signal into approximation and details using DTCWT
2. Treat the detail components using optimum thresholding rule
3. Compute the Inverse DTCWT of the approximation and treated detail coefficients to obtain the denoised signal.

The DTCWT based signal denoising technique outperforms over the DWT based denoising method, because of DTCWT's quasi shift invariant nature, and improved sensitivity to the variations in the signal.

B. Analytic Wavelet Transform based Signal Denoising Technique.

Analytic Wavelet Transform (AWT) is a member of the Complex Wavelet Transform, and it is different from the Dual Tree Complex Wavelet Transform in the implementation. The DTCWT makes use complex wavelet filters for wavelet decomposition of a signal. It is not easy to design complex filters which suits the particular signal to be decomposed, but it is easy to make a signal as the complex with the help of Hilbert Transform, and processing the signal using real wavelets i.e., Discrete Wavelet Transform (DWT).

A real valued signal $x(t)$ can be converted to analytic (complex signal) signal $z(t)$, without affecting the characteristics of the signal, using following mathematical expression.

$$z(t) = x(t) + jy(t) \dots\dots\dots (2)$$

where $y(t) = \sqrt{-1} \cdot H\{x(t)\}$, Hilbert Transform of $x(t)$. and hence, the Fourier Transform of $z(t)$, $Z(\omega) = 0$ for $\omega < 0$,

From the Fourier Transform of the expression (2), it follows that,

$$Y(\omega) = (-j \operatorname{sgn}(\omega))(X(\omega)) \dots\dots\dots (3),$$

where $X(\omega)$ and $Y(\omega)$ are the Fourier transforms of $x(t)$ and $y(t)$ respectively, and

$$\operatorname{sgn}(\omega) = \begin{cases} -1 & \text{if } \omega < 0 \\ 0 & \text{if } \omega = 0 \\ 1 & \text{if } \omega > 0 \end{cases}$$

In DTCWT, a real valued signal is processed using complex wavelet filters, whereas a complex signal is processed real valued wavelet filters in AWT, and the implementation is shown in the following Fig 3.

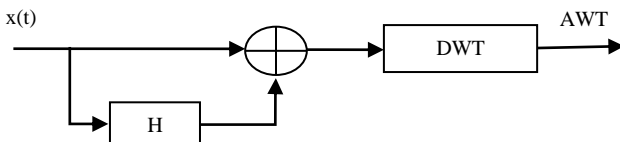


Fig 3. Implementation of AWT

The coefficients of AWT are

$$\begin{aligned} d_{\text{AWT}}(t) &= \langle x_a(t), \psi_n(t) \rangle \\ &= \langle x(t) + jH\{x(t)\}, \psi_n(t) \rangle \\ &= \langle x(t), \psi_n(t) \rangle + j \langle H\{x(t)\}, \psi_n(t) \rangle \\ &= \langle x(t), \psi_n(t) \rangle + j \langle x(t), H\{\psi_n(t)\} \rangle \\ &= \langle x(t), \psi_n(t) + jH\{\psi_n(t)\} \rangle \\ &= \langle x(t), \psi_{a,n}(t) \rangle \dots\dots\dots (4) \end{aligned}$$

where, $x_a(t)$ is the analytic form of the signal $x(t)$ and $\psi_{a,n}(t)$ is the Analytic form of Discrete Wavelet Transform $\psi_n(t)$.

The AWT based signal denoising is performed using the following algorithm.

1. Compute the Hilbert Transform of the given noisy signal $x(t)$ and express it as analytic signal, $y = H\{x\}$
2. Determine the approximation and details by computing DWT of the real and imaginary part of the analytic signal separately, $\text{DWT}\{x\}$, $\text{DWT}\{y\}$

3. Treat the detail coefficients using optimum threshold rules
4. Compute the Inverse DWTs of real and imaginary approximation and treated detail coefficients separately.
5. Find the inverse Hilbert Transform of the imaginary part of the signal, $-H\{y'\}$, y' is the denoised using.
6. The denoised signal is signal is obtained by taking the average of the signals x' and $-H\{y'\}$, where x' and y' are the denoised versions of x and y using DWT.

Here, the Analytic Wavelet Transform analyses the phase of the signal apart from magnitude of the signal. Hence, AWT based denoising technique exhibits improved performance over the DWT and same as that of DTCWT.

3. HYBRID DUAL TREE COMPLEX WAVELET BASED IMAGE DENOISING

Dual Tree Complex Wavelet Transform (DTCWT) and Analytic Wavelet Transform (AWT) are two different version of Complex Wavelet Transform, which can be used to analyze the signal in an efficient and effective manner.

The proposed technique Hybrid Dual Tree Complex Wavelet Transform (HDTCWT) combines the features of the both Dual Tree Complex Wavelet Transform (DTCWT) and Analytic Wavelet Transform (AWT). Thus, improved performance can be expected from the HDTCWT based signal denoising method over the DTCWT and AWT based denoising techniques.

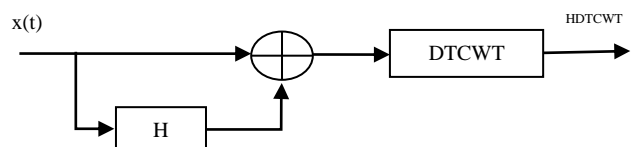


Fig 4. Implementation of HDTCWT

The implementation of HDTCWT is as shown in Fig 4. In the HDTCWT technique, the analytic form of the given signal is computed initially and the analytic signal processed using Dual Tree Complex Wavelet Transform.

The coefficients of the HDTCWT are determined in similar with the computation of the AWT coefficients as in (4). The HDTCWT wavelet coefficients are given by

$$d_{\text{HDTCWT}}(t) = \langle x_a(t), \psi_r(t) + j\psi_i(t) \rangle \dots\dots\dots (5).$$

where, $x_a(t)$ is the analytic form of the signal $x(t)$ and is given as $x_a(t) = x(t) + jH\{x(t)\}$ and $\psi_r(t)$ and $\psi_i(t)$ are the wavelet functions of real tree and imaginary tree in DTCWT.

The algorithm for HDTCWT based signal denoising technique is as follows:

1. Determine the analytic signal $x_a(t)$ of the given signal $x(t)$ with the help of Hilbert Transform.
2. Apply Dual Tree Complex Wavelet Transform (DTCWT) to both the real and imaginary parts of the analytic signal separately.

3. Treat all the details coefficients of both DTCWTs using optimum threshold rule
4. Compute the Inverse DTCWT over the filtered coefficients in order to obtain the denoised real and imaginary parts of the analytic signal.
5. Calculate the inverse Hilbert Transform of the denoised imaginary part of the analytic signal and is averaged with denoised real part of analytic signal to get overall denoised signal.

As the Hybrid Dual Tree Complex Wavelet Transform (HDTTCWT) merges the important features of both Analytic Wavelet Transform (AWT) and Dual Tree Complex Wavelet Transform (DTCWT), HDTTCWT based signal denoising technique will perform better than the DWT, AWT and DTCWT based denoising techniques.

4. RESULTS AND DISCUSSION

The performance of the DWT, AWT, DTCWT and HDTTCWT based signal denoising techniques are compared with the help of the performance metrics Signal to Noise Ratio (SNR) and Correlation Coefficient.

SNR of two signals x and y is given as

$$\text{SNR (dB)} = 10 \log_{10} \left(\frac{\sum_{i=0}^{N-1} x(i)^2}{\sum_{i=0}^{N-1} y(i)^2} \right)$$

and Correlation Coefficient $C = \frac{\sqrt{\sum_{i=0}^{N-1} (x(i)y(i))^2}}{\sqrt{\sum_{i=0}^{N-1} x(i)^2} \sqrt{\sum_{i=0}^{N-1} y(i)^2}}$

where N is the number of samples in the signals (sequences) in x or y .

In this work, two different signals, Chirp signal is considered to evaluate the performance of the different wavelet transform based signal denoising techniques discussed in this paper.

The Chirp signal is a one in which the signal frequency is varying continuously, hence the best suitable wavelet technique which senses the continuous change in the frequency will be decided. Here, original (clean) Chirp signal is considered and it is added with Additive White Gaussian Noise (AWGN) so that the resultant Noisy signal SNRs in the Ratio of -10 dB to +15 dB.

In the Fig 5, Chirp signal (original signal), and the signal corrupted with Gaussian noise (0 dB), and its denoised versions using DWT (with db8 wavelet), AWT (with db8 wavelet), DTCWT and HDTTCWT are shown in the Fig 5(a), 5(b), 5(c), 5(d), 5(e) and 5(f) respectively. Table 1(a) and (b) summarizes the SNR values and Correlation coefficients of the Chirp signal corrupted with Additive White Gaussian Noise within the range of -10 dB to +15 dB.

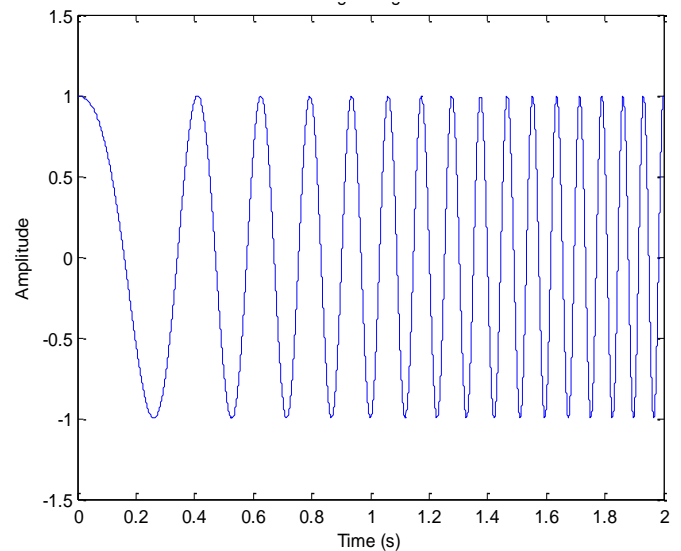


Fig 5(a) Original Signal (Chirp)

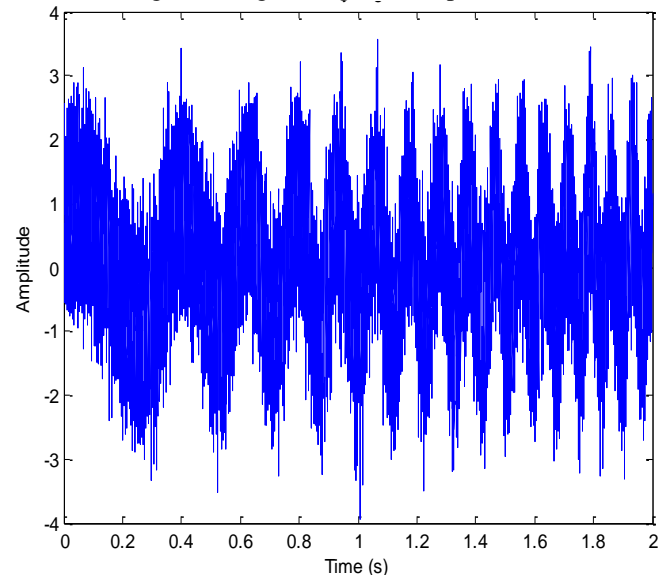


Fig 5(b) Noisy Chirp Signal (SNR = 0 dB, Correlation Coefficient = 0.7091)

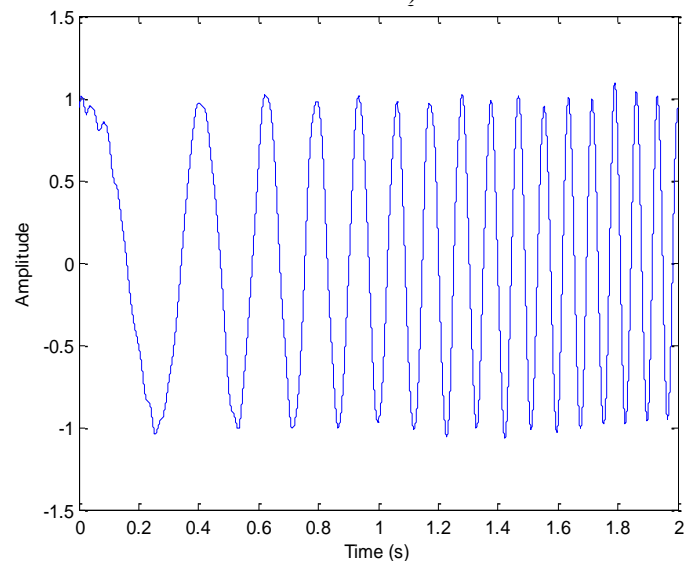


Fig 5(c) Denoised Signal using DWT (SNR = 25.4276 dB, Correlation Coefficient =0.9986)

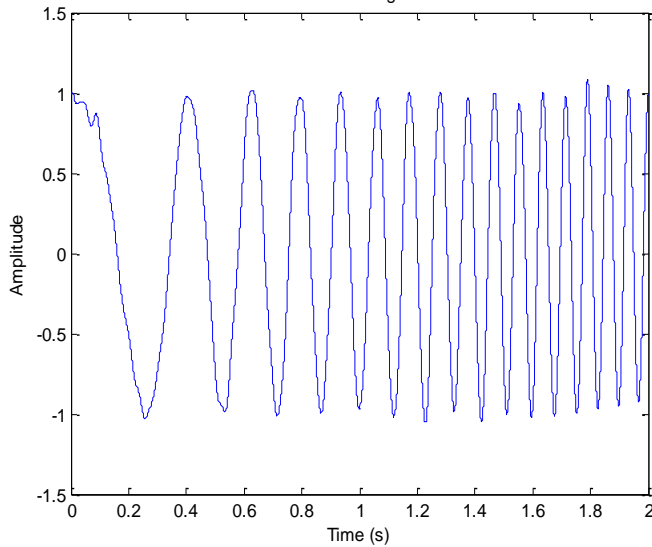


Fig 5(d) Denoised using AWT (SNR = 25.8699 dB, Correlation Coefficient =0.9987)

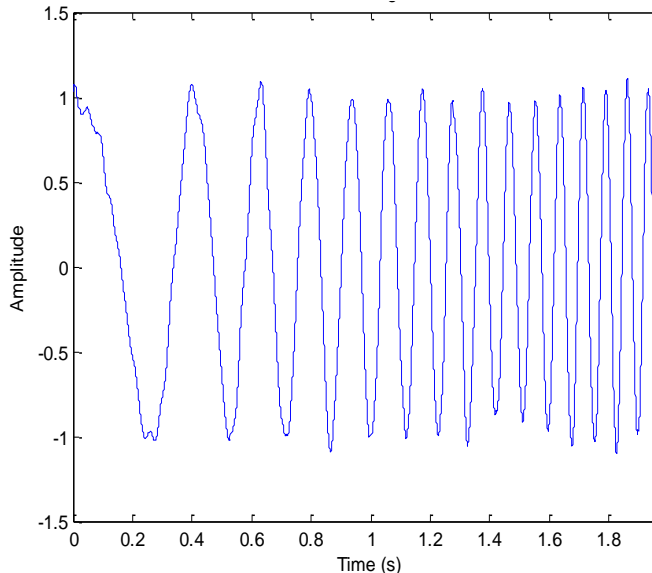


Fig 5(e) Denoising using DTCWT (SNR = 27.3986 dB, Correlation Coefficient =0.9988)

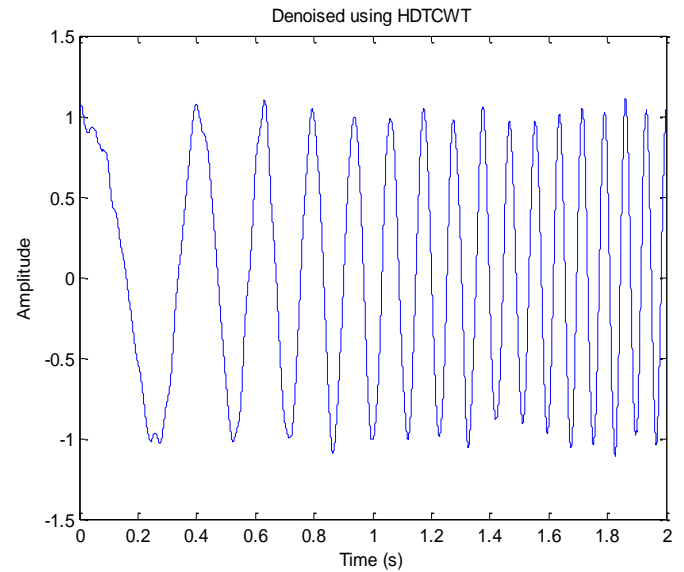


Fig 5(f) Denoised using HDTCWT (SNR = 27.4024 dB, Correlation Coefficient =0.9989)

Table 1(a) SNR (dB) Values of the Denoised Signals (Chirp) using Different Wavelet Techniques

SNR	-10	-5	0	5	10	15
DWT	8.595	20.413	25.428	30.249	34.727	38.395
AWT	9.890	20.867	25.870	30.812	35.620	40.038
DT CWT	16.081	20.997	27.399	31.999	35.985	39.987
HDT CWT	16.099	21.017	27.402	32.054	36.220	40.463

Table 1(b) Correlation Coefficient of Denoised Signals (Chirp) using Different Wavelet Techniques

SNR	-10	-5	0	5	10	15
DWT	0.928	0.996	0.999	1.000	1.000	1.000
AWT	0.947	0.996	0.999	1.000	1.000	1.000
DT CWT	0.988	0.996	0.999	1.000	1.000	1.000
HDT CWT	0.988	0.996	0.999	1.000	1.000	1.000

The graphical interpretation of the SNR and Correlation Coefficient Values of the Denoised signals are Shown in Fig 6(a) and 6(b) respectively.

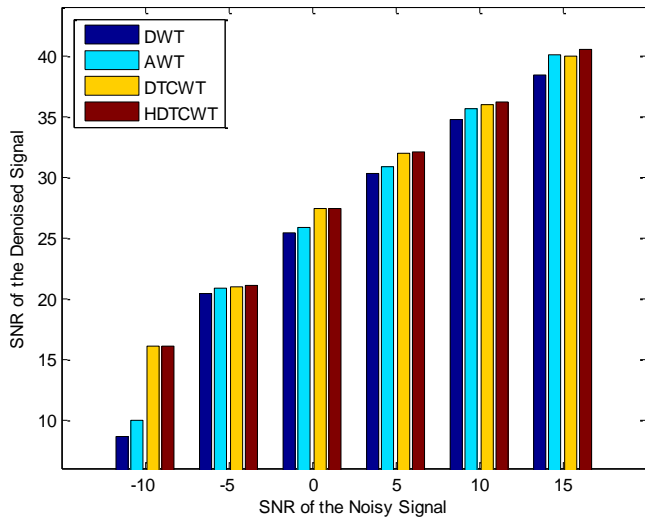


Fig 6(a) Graphical interpretation of the SNR values of denoised signal versus SNR values of Noisy Signal

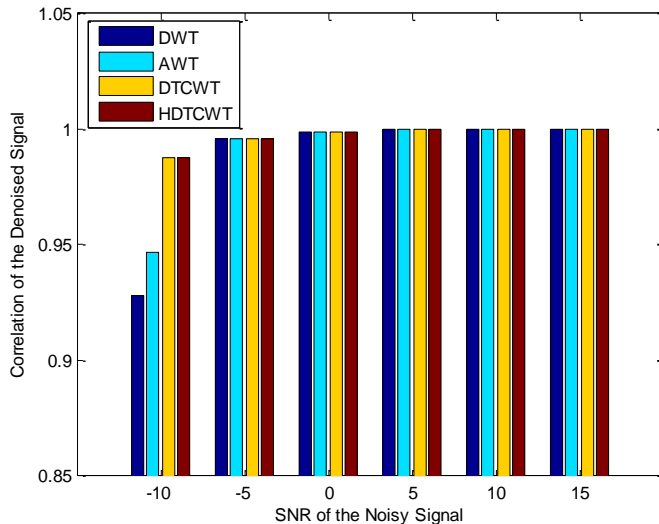


Fig 6(b). Graphical interpretation of Correlation Coefficients of Denoised Signal versus SNR values of the Noisy Signals

From the tables 1(a) & 1(b), it is obvious that Hybrid Dual Tree Complex Wavelet Transform (HDTCWT) based signal denoising technique performs better than all other wavelet techniques. Also, HDTCWT and DTCWT based denoising technique performance is far better than the DWT and AWT based denoising technique, when the noise is high ($\text{SNR} < 0$ dB). However, HDTCWT and DTCWT based techniques denoising efficiency asymptotically approach the denoising efficiency of the DWT and AWT based denoising techniques when the noise is low ($\text{SNR} \geq 10$ dB). In other words, DTCWT and HDTCWT based denoising techniques do not exhibit significant performance in denoising a signal compared to the DWT and AWT based when the noise is low ($\text{SNR} > 15$ dB).

Fig 6(a) and 6(b) shows the graphical interpretation of the SNR values of the Correlation Coefficient Values of the denoised signal with respect to the SNR of the noisy signals. It is quite obvious that both SNR of the denoised signal and Correlation between the denoised

signal and original signal are increasing with increase in the SNR of the noisy signal. Further, the Correlation coefficients reach almost unity for denoising the noisy signals with $\text{SNR} > 5$ dB. In over all, the proposed technique HDTCWT based signal denoising outperforms over the other techniques namely DWT, AWT and DTCWT based denoising techniques.

The values of the performance metrics (SNR and Correlation Coefficient) are not fixed for given SNR of the noisy signals, the performance metric values vary with the change in the characteristics of the signal.

5. CONCLUSION

In this work, the denoising efficiency of the various types of wavelet transform based denoising techniques evaluated and compared to one another with the help of the performance metrics SNR and Correlation Coefficient. From the comparison of the performance metrics, it is found that the performance of the proposed technique HDTCWT based signal denoising is better than that of the other techniques like DWT, AWT, DTCWT based signal denoising techniques. As the DWT suffers from the limitation such as shift sensitivity, lack of signal phase information, DWT based denoising technique is not as effective as complex wavelet transform based denoising techniques. AWT and DTCWT based denoising techniques are more efficient because they have overcome the limitations like shift sensitivity and absence signal phase information. Each of these techniques (AWT, DTCWT) perform denoising with their own important features, the performance of these techniques can be maximized by combining both the techniques into a single technique and the same is proved from the results.

HDTCWT based denoising technique can be applied to the different signal and image processing applications like, Biomedical Signal Processing, Radar Signal Processing etc., Though the proposed technique performs better in signal denoising, there will be always room for further improvement of the denoising capability of the technique. The further improved performance can be observed by designing wavelet filters which suits well to the given signal, and by choosing effective shrinkage technique to determine the threshold value for treating the detail coefficients of the wavelet decomposition.

REFERENCES

- [1] N. Kingsbury, "Complex Wavelets for Shift Invariant Analysis and Filtering of Signals", *Applied and Comp. Harm. Anal.* 10, 2001, 234-253.
- [2] N G Kingsbury, "A Dual-Tree Complex Wavelet Transform with improved orthogonality and symmetry properties", *Proc. IEEE Conf. on Image Processing, Vancouver, September 11-13, 2000*, paper 1429.
- [3] Iona Adam et al, "A New Implementation of Hyperanalytic Wavelet Transform", *International*

- Conference on Signals, Circuits and Systems, 2007, IEEE Explore,
- [4] I. Firoiu, C. Naornita, "Image Denoising using a New Implementation of HyperanalyticWavelet Transform",IEEE 2009
 - [5] D. Leela Rani, S. Varadarajan, M. Sadasiva, "Atmospheric Radar Signal Processing using Complex Wavelet Transforms", International Journal of Computer Applications, ISSN: 0975-8887, Vol 91, No 11, 2014, pp. 33-37
 - [6] D. Leela Rani, S.Varadarajan, M.Saasiva, "MST RadarSignal Processing using Analytic Wavelet Transform,"Proc., Second International Conference on AdvancedComputing Methodologies,(ISBN:9789351071495),GRIET, Hyderabad, pp. 353-355,21-23 November 2013
 - [7] D. Sreenivasulu Reddy et al, "A New Image Denoising Scheme using Hyperanalytic Wavelet Transfrom with an Adaptive Wiener Filter", DIGITECH: An International Journal of Computer Science and Applications. (ISSN: 2229-4597), Vol. 3, No. 6, pp. 29-36
 - [8] S. Mallat, "A Wavelet Tour of Signal Processing". Academic Press, 1999.
 - [9] M. Sadasiva et al, "A New Wiener Filter based Image Denoising Scheme using Hyperanalytic Wavelet Transform", International Conference on Emerging Trends in Electrical, Communication and Information Technologies, (ICECT 2012)
 - [10] D. L. Donoho, I. M. Johnstone, Ideal spatial adaptation by wavelet shrinkage, Biometrika, 81(3) : 425-455, 1994